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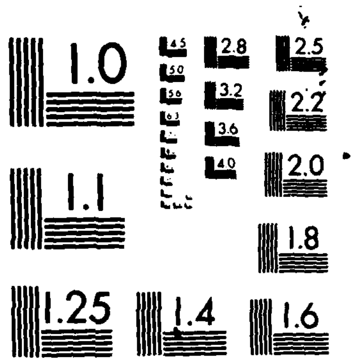

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This report is devoted to a computational method of mesh generation in arbitrary surfaces by utilizing a set of elliptic partial differential equations. These equations depend explicitly on the mean curvature and the unit normal vector of the surface in which the coordinates are to be generated. To determine the mean curvature for a given surface in global coordinates, first a piecewise least squares method is used to fit a surface through the given data points. Next, mesh generation results for various geometrically complicated shapes have been obtained to demonstrate the versatility of the proposed equations. An example of a monoclinic coordinate system with contraction in the coordinate leaving the surface has also been presented.

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Generation of Surface Grids Through Elliptic Partial Differential Equations for Aircraft and Missile Configurations

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1. INTRODUCTION

In recent times there has been much interest in the generation of spatial grids by numerical methods either by algebraic means or by solving certain partial differential equations. A collection of various methods has appeared in [1] and reference may also be made to the review articles [2], [3], and to various conference proceedings, [4,5].

This paper is directed to the problem of grid generation in a given surface by utilizing a set of elliptic equations. The proposed mathematical model has deliberately been made to depend on the formulae of Gauss, which naturally involve the partial derivatives of the Cartesian coordinates with respect to the curvilinear coordinates. With the formulae of Gauss as the basis for the proposed elliptic equations it can be stated with certainty that every smooth surface (second order differentiable) must satisfy these equations for any allowable coordinate system introduced in the surface. Work on these lines was initiated by Warsi [6-9], and an exclusive article on surface grids [10] may also be consulted. Various other contributions by using the proposed equations have been made [11-15]. An important point to be made here is that all other successful elliptic models for surface grid generation are a consequence of the proposed equations (Eq. (5)), e.g., Garon and Camarero [16] and Thomas [17].

The proposed equations (Eqs. (5)) can be used to generate the Cartesian coordinates as functions of the curvilinear coordinates when the surface has been specified either analytically or by discrete data points. Further, the boundary lines in the piece of a surface in which the coordinates are to be introduced have to be decided in advance. There is, however, no restriction whether the piece of the surface forms a simply- or multiply-connected domain.

For arbitrary shaped bodies in which only discrete data points of the surface are given, it is important to fit a global equation of the form $F(x,y,z)=0$ so as to express the mean curvature of the surface as a function of x,y,z. A method which has worked for many cases including the case of a fuselage is that of <u>overlapping piecewise least-squares</u> method. Results for some geometrically difficult cases have been presented in this paper. Also, as an application of the surface grid generation scheme reported here, the case of monoclinic coordinates has been considered.

2. NOMENCLATURE

$b_{\alpha\beta}$ = $\tilde{n}^{(\nu)}\cdot\tilde{r}_{,\alpha\beta}$; the coefficients of the second fundamental form for the surface x^{ν}=const.

D = differential operator defined in Eq. (3).

G_{ν} = $g_{\alpha\alpha}g_{\beta\beta}-(g_{\alpha\beta})^2$, where ν,α,β are cyclic.

g_{ij}	= covariant metric components.
g^{ij}	= contravariant metric components.
g	= det (g_{ij}).
J	= Jacobian determinant.
$k_I^{(\nu)}, k_{II}^{(\nu)}$	= principal curvatures at a point in x^ν=const.
L	= differential operator.
$\underset{\sim}{n}^{(\nu)}$	= unit normal vector on x^ν=const.
$\overline{P},\overline{Q}$	= control functions
$P^\alpha_{\beta\gamma}$	= control functions.
$\underset{\sim}{r}$	= rectangular Cartesian coordinates x,y,z.
R	$= G_\nu(k_I + k_{II})$.
x^i	= 3D curvilinear coordinates; $i = 1,2,3$.
x^α	= 2D curvilinear coordinates.
$x^\alpha_{(m)}$	= coordinates in successive transformations; $m = 0,1,2,...$
$X^{(\nu)}, Y^{(\nu)}, Z^{(\nu)}$	= rectangular components of $\underset{\sim}{n}^{(\nu)}$.
$T^\delta_{\alpha\beta}$	$= \frac{1}{2} g^{\sigma\delta}\left(\frac{\partial g_{\alpha\sigma}}{\partial x^\beta} + \frac{\partial g_{\beta\sigma}}{\partial x^\alpha} - \frac{\partial g_{\alpha\beta}}{\partial x^\sigma}\right)$; the surface Christoffel symbols of the second kind.
Γ^k_{ij}	$= \frac{1}{2} g^{mk}\left(\frac{\partial g_{im}}{\partial x^j} + \frac{\partial g_{jm}}{\partial x^i} - \frac{\partial g_{ij}}{\partial x^m}\right)$; the space Christoffel symbols of the second kind.
$\Delta_2^{(\nu)} x^\delta$	$= -g^{\beta\gamma} T^\alpha_{\beta\gamma}$; Beltramian.
$\nabla^2 x^i$	$= -g^{rj}\Gamma^i_{rj}$; Laplacian.
$\underset{\sim}{r}_{,\alpha}$	$= \frac{\partial \underset{\sim}{r}}{\partial x^\alpha}$.

$$\underset{\sim}{r}_{,\alpha\beta} = \frac{\partial^2 \underset{\sim}{r}}{\partial x^\alpha \partial x^\beta}.$$

$$\underset{\sim}{r}_\xi = \frac{\partial \underset{\sim}{r}}{\partial \xi}.$$

$$\underset{\sim}{r}_{\xi\eta} = \frac{\partial^2 \underset{\sim}{r}}{\partial \xi \partial \eta}.$$

Other Rules:

1. Repeated lower and upper indices will always imply summation over the range of index values.

2. Greek indices except ν take values (1,2) or (2,3) or (3,1), while the Latin indices take values 1,2,3.

3. If an index is enclosed in parentheses then the summation convention (rule 1) is inapplicable.

3. THE MATHEMATICAL MODEL

The mathematical basis of the present formulation along with the derivation of the elliptic equations for the generation of surface grids has been discussed in various publications, [6] - [10]. The main steps have, however, been summarized below.

Usually the main aim of grid generation is to generate the rectangular Cartesian coordinates on the curvilinear parametric coordinates and hence equations must be obtained in which the curvilinear coordinates appear as the independent variables. Fortunately, the formulae of Gauss provide a direct access to those quantities in which the surface curvilinear coordinates already appear as the independent variables. If x^α (α assuming only two values) are the surface coordinates in the surface x^ν=const., then the formulae of Gauss (cf. [7,18]) are

$$\underset{\sim}{r}_{,\alpha\beta} = T^{\delta}_{\alpha\beta}\underset{\sim}{r}_{,\delta} + \underset{\sim}{n}^{(\nu)}b_{\alpha\beta}, \tag{1}$$

where all quantities appearing in (1) have been defined in Sect. 2. Inner multiplication of Eq. (1) with $G_\nu g^{\alpha\beta}$ results in the vector equation

$$D\underset{\sim}{r} + G_\nu(\Delta_2^{(\nu)}x^\delta)\underset{\sim}{r}_{,\delta} = \underset{\sim}{n}^{(\nu)}R, \tag{2}$$

where

$$\begin{aligned} D &= G_\nu g^{\alpha\beta}\partial_{\alpha\beta}, \\ R &= (k_I^{(\nu)} + k_{II}^{(\nu)})G_\nu \\ &= g^{\alpha\beta}b_{\alpha\beta}, \\ \Delta_2^{(\nu)}x^\delta &= -g^{\alpha\beta}\gamma^{\delta}_{\alpha\beta}. \end{aligned} \tag{3}$$

For the purpose of imposing a control on the distribution of coordinates, we set

$$\Delta_2^{(\nu)}x^\delta = g^{\alpha\beta}P^{\delta}_{\alpha\beta}, \tag{4}$$

where $P^{\delta}_{\alpha\beta}$ are six arbitrarily specified control functions. For a thorough discussion on the properties of $P^{\delta}_{(\alpha\beta)}$ and also on the relationship between the Beltramians $\Delta_2^{(\alpha\beta)} x^{\delta}$ and the 3D Laplacians $\nabla^2 x^{\delta}$, refer to Warsi [10], [19], and to Sect. 5 of this paper.

Substitution of Eq. (4) in Eq. (2) yields a deterministic set of the grid generation equations. As an example, if $x^3=\zeta=$const. is the surface in which one wishes to introduce the coordinates ξ and η, then the three scalar equations from Eq. (1) are

$$Lx = X^{(3)}R, \quad Ly = Y^{(3)}R, \quad Lz = Z^{(3)}R, \tag{5}$$

where

$$L = g_{22}\partial_{\xi\xi} - 2g_{12}\partial_{\xi\eta} + g_{11}\partial_{\eta\eta} + \overline{P}\partial_{\xi} + \overline{Q}\partial_{\eta},$$

$$\overline{P} = g_{22}P^1_{11} - 2g_{12}P^1_{12} + g_{11}P^1_{22},$$

$$\overline{Q} = g_{22}P^2_{11} - 2g_{12}P^2_{12} + g_{11}P^2_{22}.$$

Here

$$g^{11} = g_{22}/G_3, \quad g^{12} = -g_{12}/G_3, \quad g^{22} = g_{11}/G_3,$$

$$G_3 = g_{11}g_{22} - (g_{12})^2. \tag{6}$$

4. NUMERICAL SOLUTION

Numerical solution of Eqs. (5) has been obtained by using point and LSOR iterative schemes. The main difference between the grid generation in a flat space and in a curved surface is due to the appearance of the forcing terms on the right hand sides of Eqs. (5). The quantity R has been defined in Eq. (3) and is composed of the principal curvatures as local functions of coordinates in a surface. Thus the distinguishing features of the body geometry are contained in both R and the unit normal vector $\underset{\sim}{n}$. Before attempting to solve Eqs. (5) it is important to feed the body-geometry information either through R or $\underset{\sim}{n}$. Most of the computer runs for different shapes reported in Refs. [10] and [20] and in this paper have been conducted by prescribing R as a function of x,y,z. That is, once the equation of a surface either in the form F(x,y,z)=0 or z=f(x,y) has been established, the principal curvature is expressed as a function of x,y,z by using the formula.

$$
\begin{aligned}
k_I + k_{II} = [&(F_y^2 + F_z^2)(2F_xF_zF_{xz} - F_z^2F_{xx} - F_x^2F_{zz}) \\
&+ 2F_xF_y(F_z^2F_{xy} + F_xF_yF_{zz} - F_yF_zF_{xz} - F_xF_zF_{yz}) \\
&+ (F_x^2 + F_z^2)(2F_yF_zF_{yz} - 2F_z^2F_{yy} - F_y^2F_{zz})]/P^3F_z^2, \qquad (7)
\end{aligned}
$$

where

$$P^2 = F_x^2 + F_y^2 + F_z^2.$$

As has been noted in [10], if at any point $F_z = 0$, then the other form of Eq. (7) is obtained by interchanging x by y, y by z, and z by x. The information supplied by (7) is enough, since both G_3 and $\underset{\sim}{n}$ are calculated by the iterative scheme itself.

Much of the present authors' efforts have been directed toward obtaining a numerical scheme which can be used for arbitrarily shaped surfaces so as to fit a function F(x,y,z)=0 based on the given discrete surface data. Some success has been achieved by using <u>overlap-piecewise</u> least squares method. In each piece a second degree polynomial in x,y,z has been used. Many known surfaces of second degree, e.g., ellipsoid, hyperboloids etc. have been duplicated by using this method. This method has also been used to duplicate an airplane fuselage for which discrete x,y,z values were available. Most of the distortion appears in the canopy part of the fuselage, though the coordinates generated by Eqs. (5) seem to be smooth. Figure 5 demonstrates the slight distortion in the body geometry and the generated grids on the fuselage. Figures 1-5 show the capability of the grid generation equations (Eq. (5)) for some difficult cases in each of which the analytical form of the equation F(x,y,z)=0 is known beforehand. For grids generated on other known surfaces refer to [10] and [20].

5. BELTRAMIAN AND LAPLACIAN APPROACHES

For the purpose of comparison of the surface grid generation model (Eq. (5)) with the surface grid generation by using the Laplace/Poisson equations in 3D at a surface, Warsi in [10] and [19] has established the relevant relationships between the two approaches. This analysis shows that if the transverse coordinate (the coordinate going out of the surface) is orthogonal to the surface then the inverted form of the 3D Laplace/Poisson equations and Eqs. (5) are the same. This result does not demand that the transverse coordinate, besides being orthogonal to the surface, should also be a line of zero curvature, [17].

Let ξ and η be any parametric coordinate in a surface with ζ being orthogonal to the surface. Then as shown in [10] and [19], the relations between the Beltramians and the Laplacians are as follows:

$$\Delta_2\xi = \nabla^2\xi - \frac{\Gamma^1_{33}}{g_{33}},$$

$$\Delta_2\eta = \nabla^2\eta - \frac{\Gamma^2_{33}}{g_{33}}.$$

Further

$$\nabla^2\zeta = -\frac{\Gamma^3_{33}}{g_{33}} - \frac{k_I^{(3)}+k_{II}^{(3)}}{\sqrt{g_{33}}},$$

and

$$k_I^{(3)} + k_{II}^{(3)} = -\frac{1}{2\sqrt{g_{33}}\,G_3}\frac{\partial G_3}{\partial\zeta},$$

$$G_3 = g_{11}g_{22} - (g_{12})^2. \tag{8}$$

6. COORDINATE CONTROL

The specification of the Beltramians in the form of Eq. (4) implies that the control functions $P^{\delta}_{\alpha\beta}$ have been chosen a'priori by the user. In all of the computer runs shown in Figures 1-5, we have taken $P^{\delta}_{\alpha\beta}=0$ for all the relevant index values. One case in which

$$P^{1}_{\alpha\beta} = 0, \qquad P^{2}_{11} = P^{2}_{12} = 0,$$

and P^{2}_{22} is a specified function of η (refer to [10], Eq. (4.6)) is shown in Fig. 6 which amply demonstrates the role of the control functions.

Looking at the control functions as coordinate redistribution functions, it has been shown by Warsi [10], [19] that for two successive coordinate systems denoted as $x^{\alpha}_{(m-1)}$ and $x^{\alpha}_{(m)}$, then the $P^{\delta}_{\alpha\beta(m)}$ can be expressed in terms of $P^{\delta}_{\alpha\beta(m-1)}$ as

$$P^{\delta}_{\alpha\beta(m)} = -T^{\delta}_{\alpha\beta(m)} + [P^{\sigma}_{\varepsilon\mu(m-1)} + T^{\sigma}_{\varepsilon\mu(m-1)}] \frac{\partial x^{\delta}_{(m)}}{\partial x^{\sigma}_{(m-1)}} \frac{\partial x^{\varepsilon}_{(m-1)}}{\partial x^{\alpha}_{(m)}} \frac{\partial x^{\mu}_{(m-1)}}{\partial x^{\beta}_{(m)}}, \tag{9}$$

where

$$P^{\delta}_{\alpha\beta(0)} = 0$$

for all relevant values of δ, α, β.

7. GENERATION OF NON-RECTANGULAR COORDINATES

In some problems it is desirable to generate non-rectangular coordinates on curvilinear coordinates in the same surface. The non-rectangular coordinates can be spherical, cylinderical or any general coordinates in the surface. The formulation of the pertinent equations follows directly in a simple way from Eqs. (5) and has been fully described in [1], p. 248. Refer also to [13]-[14].

The equations in this case can concisely be written as

$$L\underset{\sim}{u} = J_3^2 \Delta_2 \underset{\sim}{u} \tag{10}$$

where $\underset{\sim}{u} = (u,v)$ and

$$L = a\partial_{\xi\xi} - 2b\partial_{\xi\eta} + c\partial_{\eta\eta} + J_3^2\,(P\partial_\xi + Q\partial_\eta).$$

For an expanded form of the coefficient a,b,... refer to [1]. Here (u,v) are the non-rectangular coordinates while ξ,η are the desired coordinates. Figure 7 demonstrates the use of Eq. (10) in obtaining the coordinates on the surface of an ellipsoid forming a doubly-connected region.

8. MONOCLINIC SURFACE-ORIENTED COORDINATES

A monoclinic coordinate system is that in which the transverse or the outgoing coordinate from the surface is rectilinear and orthogonal to the surface; the other two coordinates which are in the surface can be either orthogonal or non-orthogonal. The capability of surface grid generation technique discussed in Sections 3 and 4 coupled with the orthogonal and rectilinear nature of the third coordinate provides a simple method of obtaining 3D grids. An immediate use of such coordinates is in the numerical solution of the Navier-Stokes and the boundary layer equations. Coordinate systems of the type under discussion have been considered by Hirchel [21] and more recently by Lee [22]. The essential mathematics is simple if use is made of tensor algebra.

Let a coordinate system $x^{\alpha}(x^1=\xi,\ x^2=\eta)$ has already been generated in a given surface by the method described in Sections 3 and 4. We now consider a neighboring surface parallel to the original surface and denote the position vector of any point of the parallel surface by $\bar{\underset{\sim}{r}}$. Then

$$\bar{\underset{\sim}{r}} = \underset{\sim}{r}(x^{\alpha},\zeta) = \underset{\sim}{r}(x^{\alpha}) + \phi(\zeta)\underset{\sim}{n}(x^{\alpha}). \tag{11}$$

where ζ is the outgoing coordinate from the original surface on which $\zeta=0$, and ϕ is an arbitrary (user specified) function of ζ. Using the obvious equations

$$\underset{\sim}{r}_{,\alpha} \cdot \underset{\sim}{n} = 0, \qquad \underset{\sim}{n} \cdot \underset{\sim}{n}_{,\alpha} = 0, \qquad \bar{\underset{\sim}{n}} \cdot \bar{\underset{\sim}{n}}_{,\alpha} = 0,$$

we conclude that $\bar{\underset{\sim}{n}}=\underset{\sim}{n}$, which is the condition of parallel surfaces. After some tensor algebra, the geometric quantities of the parallel surface are, (cf. [22]),

$$\bar{g}_{\alpha\beta} = (1 - K\phi^2)g_{\alpha\beta} - \{2 - (k_I + k_{II})\phi\}\phi b_{\alpha\beta}$$

$$\bar{b}_{\alpha\beta} = \{1 - (k_I + k_{II})\phi\}b_{\alpha\beta} + K\phi g_{\alpha\beta},$$

$$\bar{K} = K\{1 - (k_I + k_{II})\phi + K\phi^2\}^{-1}, \tag{12}$$

where an overhead bar is used for quantities in the parallel surface and K is the Gaussian curvature.

Using the preceding algorithm a series of parallel surfaces can be constructed starting from the data of the given surface. Each surface, starting from the first parallel surface, is obtained algebraically by using Eqs. (12). Lee [22] has taken

$$\phi(J) = \frac{(J-1)}{J_{m-1}} k^{J-JM}$$

where J=1 corresponds to $\zeta=0$, JM=maximum ζ-steps, and k>1 is a chosen constant. Figure 8 demonstrates the surfaces for a piece of an ellipsoidal surface.

9. CONCLUSIONS

The proposed set of elliptic equations for the generation of surface meshes (Eqs. (5)) have been solved in many cases. In each case the resulting coordinate lines are sufficiently smooth. An important aspect of the algorithm is to specify the equation of the surface $F(x,y,z)=0$ based on the discrete input data for a surface. This function must be differentiable at least to the second order for it to be used on the right hand side terms of Eq. (5). Though we have used a least squares aproach, our research in this area is still continuing.

REFERENCES

1. THOMPSON, J.F., WARSI, Z.U.A., and MASTIN, C.W., Numerical Grid Generation: Foundations and Applications, North-Holland, Amsterdam, 1985.

2. THOMPSON, J.F., WARSI, Z.U.A., and MASTIN, C.W., 'Boundary-Fitted Coordinate Systems for Numerical Solution of Partial Differential Equations', Journal of Computational Physics, 1982, 47, pp. 1-108.

3. EISEMAN, P.R., 'Grid Generation for Fluid Mechanics Computations', Ann. Rev. Fluid Mech., 1985, 17, pp. 487-522.

4. THOMPSON, J.F. (Ed.), Numerical Grid Generation, North-Holland, Amsterdam, 1982.

5. GHIA, K.N., and GHIA, U., (Eds.), Advances in Grid Generation, ASME Pub. No. FED-Vol. 5, 1983.

6. WARSI, Z.U.A., 'A Method for the Generation of General Three-Dimensional Coordinates Between Bodies of Arbitrary Shapes', Mississippi State University, Engineering and Industrial Research Station, Report MSSU-EIRS-80-7, 1980.

7. WARSI, Z.U.A., 'Tensors and Differential Geometry Applied to Analytic and Numerical Coordinate Generation', Ibid, MSSU-EIRS-81-1, 1981.

8. WARSI, Z.U.A., 'Basic Differential Models for Coordinate Generation', in Numerical Grid Generation, Ed. Thompson, J.F., 1982, pp. 41-77.

9. WARSI, Z.U.A., 'A Note on the Mathematical Formulation of the Problem of Numerical Coordinate Generation', Quart. Appl. Math., 1983, 41, pp. 221-236.

10. WARSI, Z.U.A., 'Numercial Grid Generation in Arbitrary Surfaces Through A Second-Order Differential-Geometric Model', Journal of Computational Physics, 1986, 63, in press.

11. WARSI, Z.U.A., and ZIEBARTH, J.P., 'Numerical Generation of Three-Dimensional Coordinates Between Bodies of Arbitrary Shapes', in Numerical Grid Generation, Ed. Thompson, J.F., 1982, pp. 717-728.

12. WARSI, Z.U.A., 'Generation of Three-Dimensional Grids Through Elliptic Differential Equations', VonKarman Institute Lecture Series on Computational Fluid Dynamics, 1984-04.

13. TIARN, W.N., MS Thesis, Mississippi State University, Dec. 1983.

14. ZIEBARTH, J.P., Ph.D Dissertation, Mississippi State University, Dec. 1983.

15. ZIEBARTH, J.P., and WARSI, Z.U.A., 'Computer Simulation of Three-Dimensional Grids', Society of Computer Simulation, San Diego, Cal., Feb. 1984.

16. GARON, A., and CAMARERO, R., 'Generation of Surface-Fitted Coordinate Grids', in Advances In Grid Generation, Eds. Ghia, K.N., and Ghia, U., 1983.

17. THOMAS, P.D., 'Construction of Composite Three-Dimensional Grids From Subregion Grids Generated by Elliptic System', AIAA Paper No. 81-0996.

18. STRUIK, D.J., Lectures on Classical Differential Geometry, Addison-Wesley, Reading, Mass., 1950.

19. WARSI, Z.U.A., 'A Synopsis of Elliptic PDE Models for Grid Generation', Appl. Math. and Comput., 1986, in press.

20. WARSI, Z.U.A., 'Numerical Grid Generation Through Second Order Differential Geometric Models', 11th IMACS World Congress, Oslo, Norway, August 1985.

21. HIRSCHEL, E.H., 'Boundary-Layer Coordinates on General Wings and Fuselages', Z. Flugwiss., 1982, 6, pp. 194-202.

22. LEE, J.H., MS Thesis, Mississippi State University, Dec. 1985.

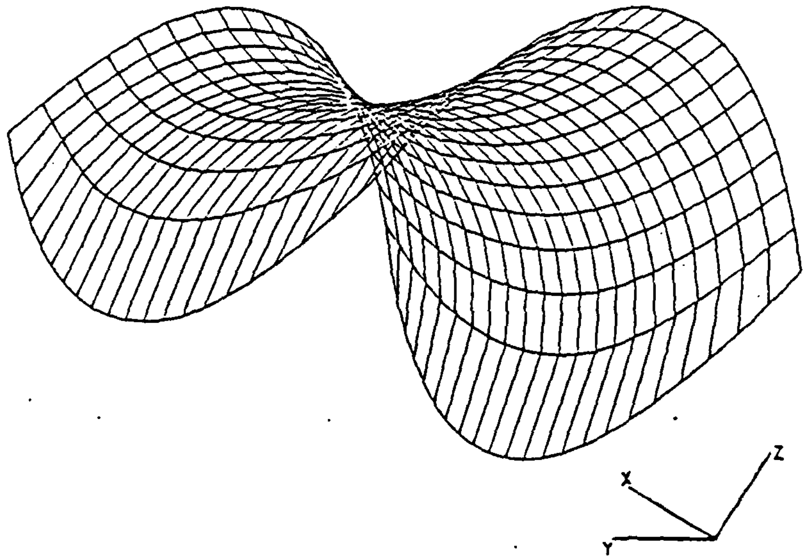

Figure 1. Coordinates on a hyperbolic paraboloid.
$z = x^2 - y^2; \quad -1 \le x \le 1, \quad -1 \le y \le 1.$

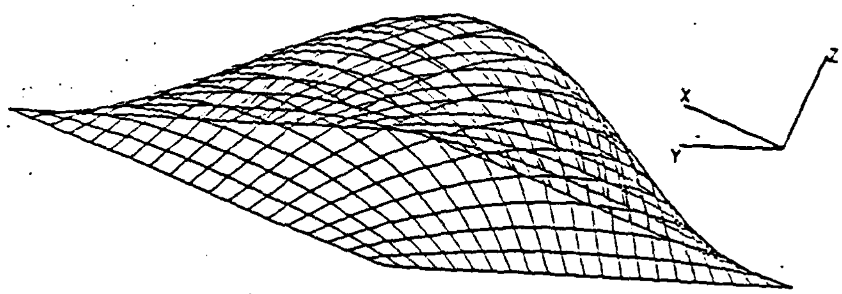

Figure 2. Coordinates on the surface:
$z = h \sin \frac{\pi x}{a} \sin \frac{\pi y}{b}, \; a = b = 1,$
$h = 0.5, \quad 0 \le x \le 1, \quad 0 \le y \le 1.$

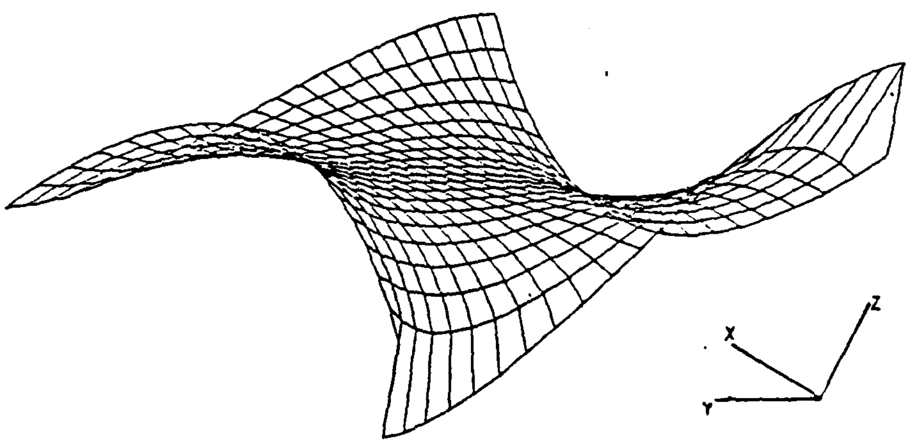

Figure 3. Coordinates on a monkey saddle.
$z = y^3 - 3x^2 y; \quad -1 \le x \le 1, \quad -1 \le y \le 1.$

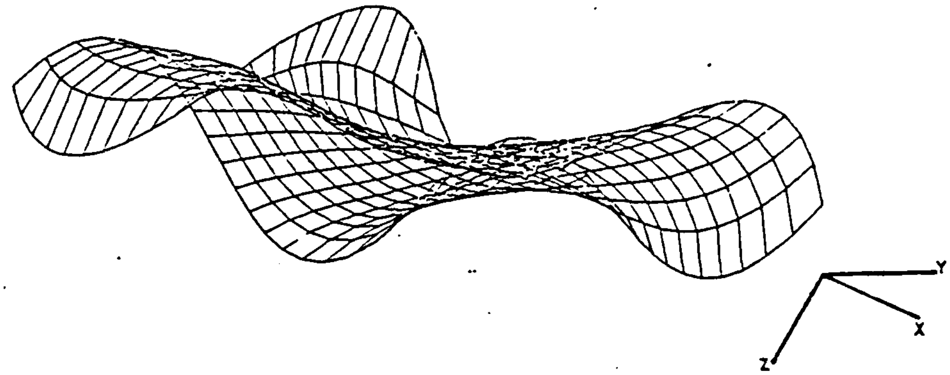

Figure 4. Coordinates on a surface having a saddle point of the higher type:
$z = xy(x^2 - y^2)$, $-1 \leq x \leq 1$, $-1 \leq y \leq 1$.

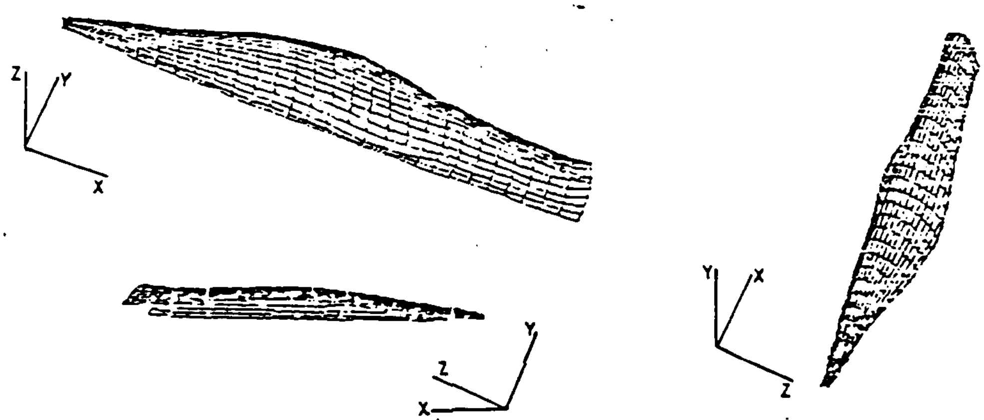

Figure 5. Three views of coordinates on a fighter plane fuselage.

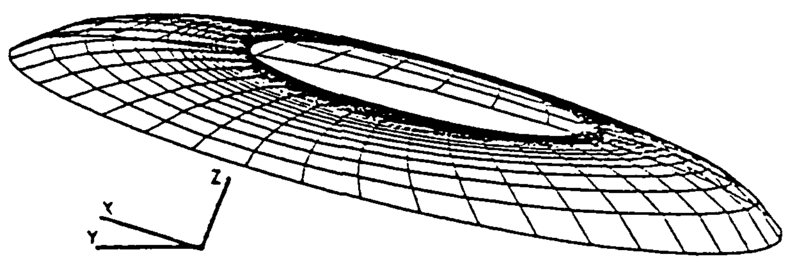

Figure 6. Example of coordinate contraction in a doubly-connected region on the surface of an ellipsoid.

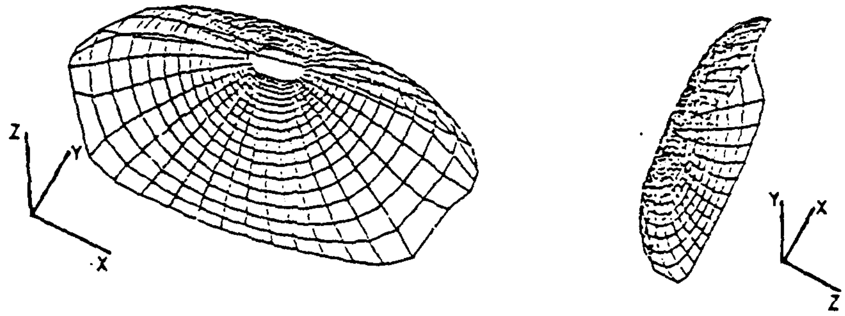

Figure 7. Coordinates in the doubly-connected region on an ellipsoid by using Eqs. (10).

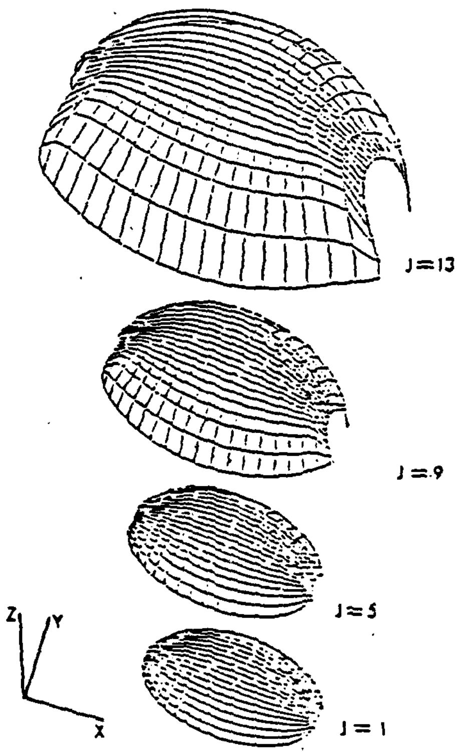

Figure 8. Monoclinic coordinates. J = 1 is the basic ellipsoidal surface in which the coordinates have been generated through using Eqs. (5), while J = 5, 9, 13 are the parallel surfaces.

Publications under Grant AFOSR-85-0143

1. "Numerical Grid Generation in Arbitrary Surfaces Through a Second-Order Differential-Geometric Model", by Z.U.A. Warsi, Accepted in J. Comp. Physics.

2. "A Synopsis of Elliptic PDE Models for Grid Generation", by Z.U.A. Warsi, Accepted in Applied Math. and Comp.

3. "Generation of Surface Coordinates by Elliptic Partial Differential Equations", by Z.U.A. Warsi, NASA Conf. Publ. 2379, June (1985).

4. "Numerical Grid Generation Through Second Order Differential-Geometric Models", by Z.U.A. Warsi, and W.N. Tiarn, 11th IMACS World Congress, Oslo, Aug. (1985).

5. "Generation of Three-Dimensional Coordinates by Using the Method of Parallel Surface", by J.H. Lee, MS Thesis, Mississippi State University, Dec. (1985).

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